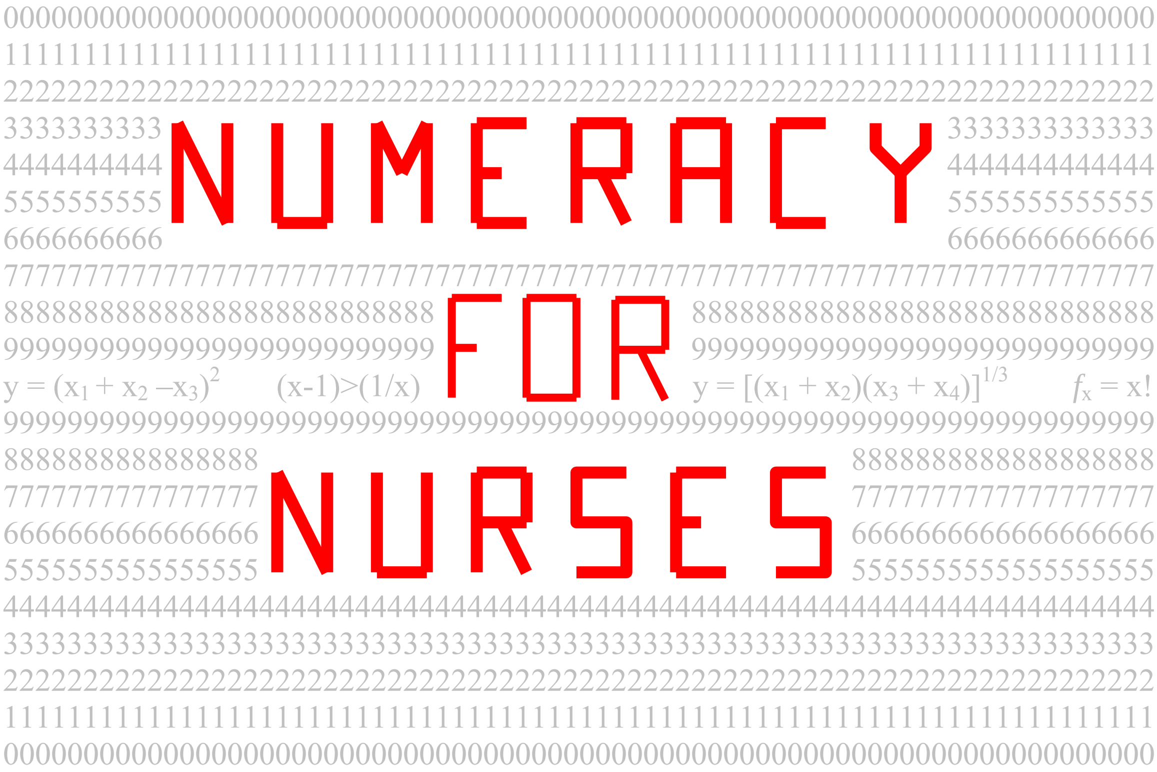

NUMERACY FOR NURSES

A Review of Fundamental Concepts in Mathematics

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About the Author

David Birnbaum earned an undergraduate degree from the University of California at Berkeley (1969: zoology), a Master of Public Health degree from the University of Minnesota School of Public Health (1974: institutional environmental health & safety), and an interdisciplinary PhD from the University of British Columbia (1992: epidemiology, statistics, medical microbiology & infectious diseases). He has held appointments as a clinical assistant professor and subsequently adjunct professor in the Department of Health Care and Epidemiology at the University of British Columbia, a lecturer at UBC's School of Nursing, a sessional instructor in the Schools of Public Administration and Business Administration at University of Victoria, is proprietor of Applied Epidemiology, and has decades of infection control experience working in the United States and Canada. He has developed knowledge-based software products for the hospital epidemiology market; courses in epidemiology, research methods and statistics for professionals in nursing, public health, infection control, health care quality improvement, hospital epidemiology, public and business administration; and consulted to various professional and government organizations. Dr. Birnbaum's research and other publications have appeared in epidemiology, microbiology and quality improvement journals. As a member of the Society for Healthcare Epidemiology of America (SHEA), he served as its designated liaison to the Health Care Division of the American Society for Quality (ASQ) as well as being a section editor for SHEA's journals, and chaired an international, interdisciplinary SHEA/ASQ-HCD Health Care Quality Improvement Special Interest Group.

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Numeracy for Nurses

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Math Fundamentals for Health Care Professionals

Preface

There are two reasons why health care professionals, especially nurses, need more than just rudimentary arithmetic capability.

First, math errors can and do kill patients. A review of fatal adverse drug events reported to the U.S. Food and Drug Administration from 1993-1998 reveals that 13% resulted from miscalculation of dosage.[1] Miscalculation was the third of seventeen leading causes, following performance deficit (such as administering an IV injection when the intended route was IM) and knowledge deficit (such as failure to apply reasonable practice standards). This is not a new concern! A 1979 study of practicing medical, nursing and pharmacy personnel determined that "one of every 12 doses computed by 95 registered nurses contained an error that would result in the administration of an amount that was ten times higher or lower than the dose ordered. The error rate was no different for experienced or inexperienced nurses… Experienced nurses tended to be more certain, although wrong, in their judgment when compared to inexperienced nurses."[2] A 1995-1996 study in a tertiary care teaching hospital found "errors in calculation or stating of dosage" to be twice as common as all "errors stated on order" and over three times as common among errors rated as potentially serious or severe.[3]

Second, health care professionals need a conceptual, not just mechanical, understanding of basic mathematics because safety cannot be proven nor quality of performance improved unless it can be measured. Whether qualitative or quantitative, meaningful measurements and analysis of those measurements requires selection of appropriate mathematical methods. Thoughtful selection of methods to summarize, analyze, and display, as well as ability to correctly interpret data displays, requires conceptual understanding. Unfortunately, there is reason to believe that many students in our programs have weak knowledge of fundamental mathematics. Studies conducted 20-30 years ago suggest that a high proportion of nursing students were deficient in basic mathematical computations and algebra skills.[4, 5] Recent experience here at UBC also suggests that many nursing students lack sufficient "numeracy" to be ready to study statistics and health research data analysis methods beyond a superficial level.

We therefore have prepared a problem set of questions for self-assessment together with this review resource to help ensure your future success. Use is entirely voluntary, but if you aren't familiar with the fundamental concepts and symbolic language reviewed in this resource then you may find yourself not ready to keep up with coursework in biostatistics or public health statistics, data analysis, and epidemiology because the mathematical "language" underlying these tools for critical appraisal will seem so foreign. If you know your math background is weak, try the review and then the self-assessment questions in its last section. If you aren't sure, skip to section 5 now and see if you can readily solve the self-assessment questions first. Answers to the self-assessment questions are available in an appendix.

<u>Literature Cited in Preface:</u>

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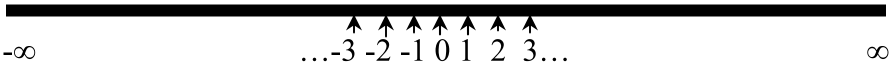
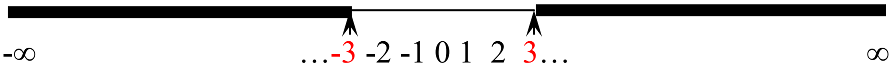
1. Representing Quantities as Numbers

You've worked with numbers all of your life, so it might seem silly that this review starts with something as simple as the concept of numbers. However, by making the concept graphically simple, this beginning sets the stage to make more complex concepts easier.

At the root of the concept of numbers is **the Number line**. This line is centered on zero and stretches from the largest negative whole number possible (negative infinity, -∞) toward the largest positive whole number possible (positive infinity, ∞):



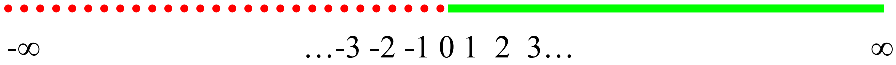
Numbers on that line represent ***magnitude*** (e.g. 3 is three units more than 0) and ***direction*** (e.g. +3 is three units to the right, -3 is three units to the left of 0). The difference between any two numbers is simply the distance between them. For example, the difference between 3 and 1, written as 3-1, obviously is 2 units. Similarly, the difference between 3 and –3, written 3-(-3), clearly is six units:



It looks obvious presented this way, so it should be just as clear in a later lesson when we use letters to represent numeric ***variables***. Just as 3 minus –3 equals 6 because we're adding the distance between 0 and +3 to the distance between 0 and –3, we'll later do this with symbols to get expressions like $x - (-x) = x + x = 2x$.

Real, Rational & Irrational Numbers

The ***integers*** we encounter every day, positive whole numbers on the right end of the number line, are known as the ***natural numbers***.



More formally, the natural numbers include all positive whole numbers (positive integers). Negative numbers are not natural numbers. Some mathematicians include 0 among the natural numbers, others don't. A reason for not including 0 is that the so-called ***rational numbers*** consist of all the natural numbers and fractions formed from them; 0 itself is considered one of the rational numbers. Since division by 0 is not possible, the rational numbers are those that can be expressed as a fraction formed by two natural numbers and then given a + or - sign. Thus, magnitudes between positive or negative integers on the number line can be formed by rational number fractions like ½, ¾, etc. Still other points between integers on the number line are the ***irrational numbers***. These are the magnitudes that cannot be expressed exactly by dividing one integer into another (examples include the square root of 2, the value of pi, etc.). All these rational and irrational numbers constitute the ***real numbers***. Values that cannot be expressed in this manner are not real numbers (e.g. the square root of –2 is not a real number).

Right about now, you might be thinking that these examples look pretty obvious and wondering why you need to know about rational, irrational, or real numbers. The next lessons switch to different numeric bases and introduce algebraic notation – the concepts and names we've just reviewed are the basics that provide a key to understanding what comes next.

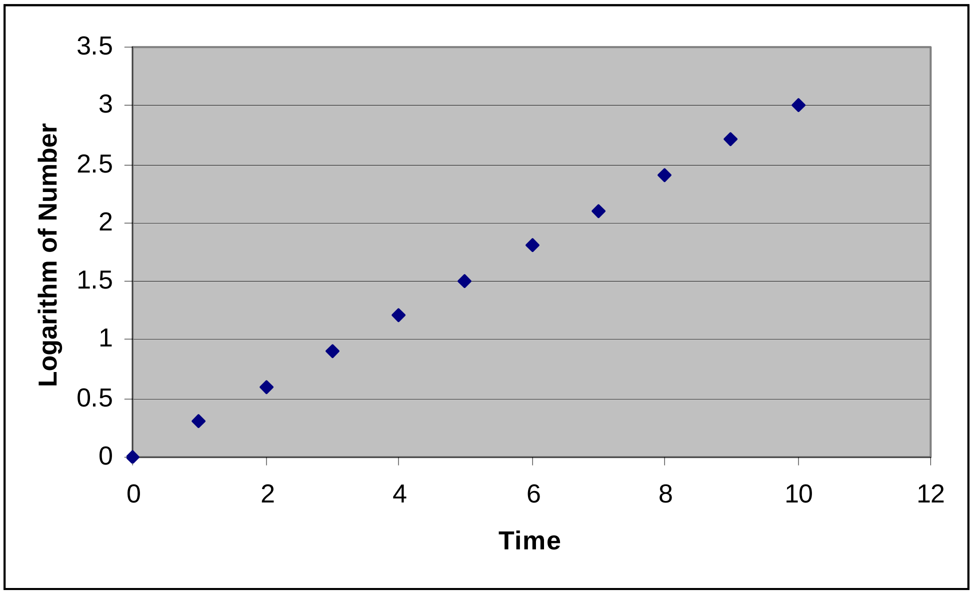
Base 10 & other bases

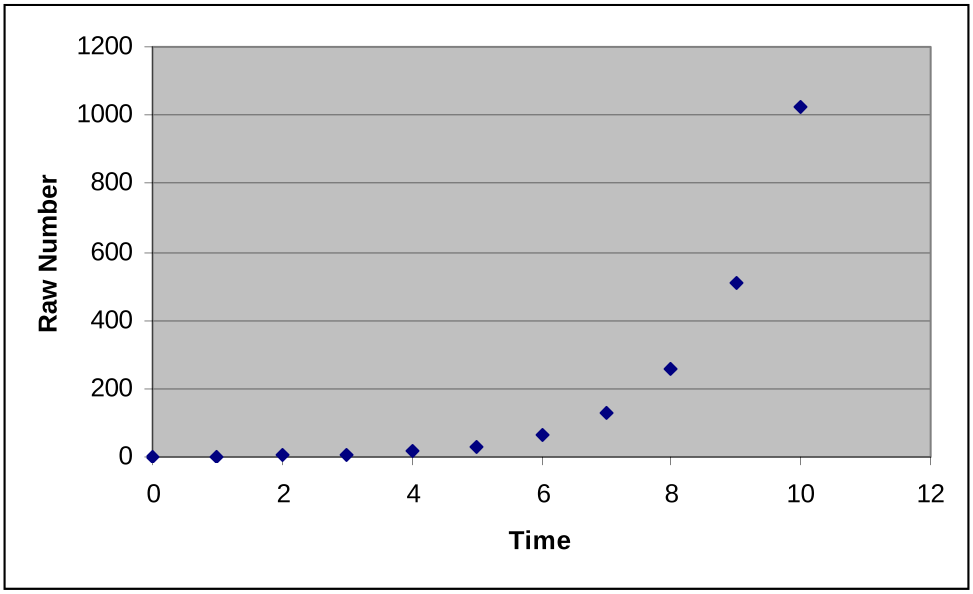
We know from years of experience that 10 means ten. But does it always? Our discussion of the number line talked about "magnitudes" and "units" represented by numbers. Perhaps because ancestors who developed our system of numerals had ten fingers on which to count, the most common number system we encounter is a "base 10" system. Adults seeing numbers like 150 don't think in terms of "0 ones, 5 tens, and 1 hundreds" (which is the way children are taught to decode the value). In a base ten system, the right-hand numeral represents increases in units of 1 from 0-9, the next numeral to the left represents advances in units of 10 from 10-90, the next numeral over in units of 100 from 100-900, and so forth. In base 10, 10 does mean a value of ten.

But base 10 isn't the only number system. Computers, for example, work in base 2. The number 11 represents a value of eleven in our usual base 10 system, but 11 represents a value of three in base 2. Computers essentially store information in a bank of many switches that each can be on or off (thus two states). The right-hand digit in a binary (base 2) system represents 0 or 1 depending on whether that switch is off (0) or on (1); beyond the maximum value that position can represent, the next position to the left represents the next largest integer value. Thus, in a base 2 number system, one is represented by 01, two by 10, three by 11, four by 100, and so on. This is still consistent with the ideas of the number line, but simply changes the magnitudes represented by numbers on that line.

Base 2 and base ***e*** (which is a number just a bit larger than 2) frequently are used in calculations related to population growth and death rate projections. Instead of the population growing or shrinking by the same absolute amount in every time period, these calculations consider growth or shrinkage by the same relative amount in each period (e.g. with base 2, going from 001 to 010 to 100 represents doubling rather than adding a specific number of members in each interval). A common way to express extremely large changes in magnitude, as well as to portray constant relative rate of change on graphs, is use of ***logarithms***. Logarithms are simply the exponent to which a base must be raised to produce a value. For example, the logarithm of 8 in base 2 is 3 (since 2^3 which is mathematical short-hand for 2 times 2 times 2 equals 8). Similarly, the logarithm of 1000 in base 10 is 3 (since 10^3 equals 10 times 10 times 10, thus 1000).

So 10 means ten units in a base 10 number system. But it also can mean two units in a base 2 system, or 10,000,000,000 units if the number is a logarithm… We're seeing the same ***numerals***, but they can be expressing different magnitudes. Why would any health care professional need to know this? There are several reasons. First, you probably see graphs drawn on ***semi-log scales*** in the journals, reports and news you encounter. You also likely see the same information drawn on linear (also known as arithmetic) scales. Values like the amount spent annually on school or health service funding, for example, can be drawn on an arithmetic scale graph to show that the absolute amount government provides has been rising more and more every year. However, graphed on semi-log scale, the same data can reveal that the rate of increase has been getting smaller and smaller over those same years… Similarly, data can be plotted in different ways to show the number of cases during outbreaks of disease. Which of these two graphs do you think would excite people the most:





In fact, they both show the same data (which has a constant rate of change over time: doubling in each time period, starting with 1 case and doubling to reach 2, 4, 8, 16, 32, 64, 128, 256, 512 and 1024 at the end of successive time periods). The top graph reveals that the rate of change remains constant (if a reader understands what the vertical logarithmic scaling means); the bottom graph shows the same data but can lead naïve readers to believe the rate of increase is increasing rapidly as time progresses. Ever wonder why many people are slow to respond to brewing situations that others recognized early, or why those same people over-react late? Perceptions (and misperceptions) created by graphs like the top versus bottom one might be part of the reason! “Numeracy” is the type of literacy that helps people understand what really is revealed or concealed, represented or misconstrued, in pictures like these.

Another reason to understand numeracy at this deeper level is preparation for graduate or post-graduate level courses in data analysis. Whether you produce or just read original research reports, understanding the statistical methods employed requires more than just mechanically memorizing formulae. Stronger numeracy knowledge gives greater chance of success in such courses, greater likelihood of competence at critical appraisal for reviewing research papers, and thus makes it easier to decide correctly whether to apply methods that new research suggests might be beneficial in your practice setting.

Yet another reason, for those with high-school aged children, is simply being able to keep up with your children when they’re looking for help with their math homework. While this study guide offers health service application examples throughout, most of the concepts reviewed appear in the high school grades 9-12 math curriculum.

Practice Problems

1. Which quantity is bigger, +3 or –3?

2. What’s the difference between an integer and a rational number?

3. Can the value 0 be expressed as a logarithm?

4. Public health professionals rely on various displays of disease occurrence data to determine whether outbreaks may be starting. The following figure from an issue of *Morbidity & Mortality Weekly Report* portrays the number of cases of select diseases reportable under public health regulations (hepatitis A, B, and C; legionellosis; measles; meningococcal infections; mumps; pertussis; and rubella) during the then-current 4 week period relative to prior periods. Why does this figure use logarithmic rather than arithmetic scaling?

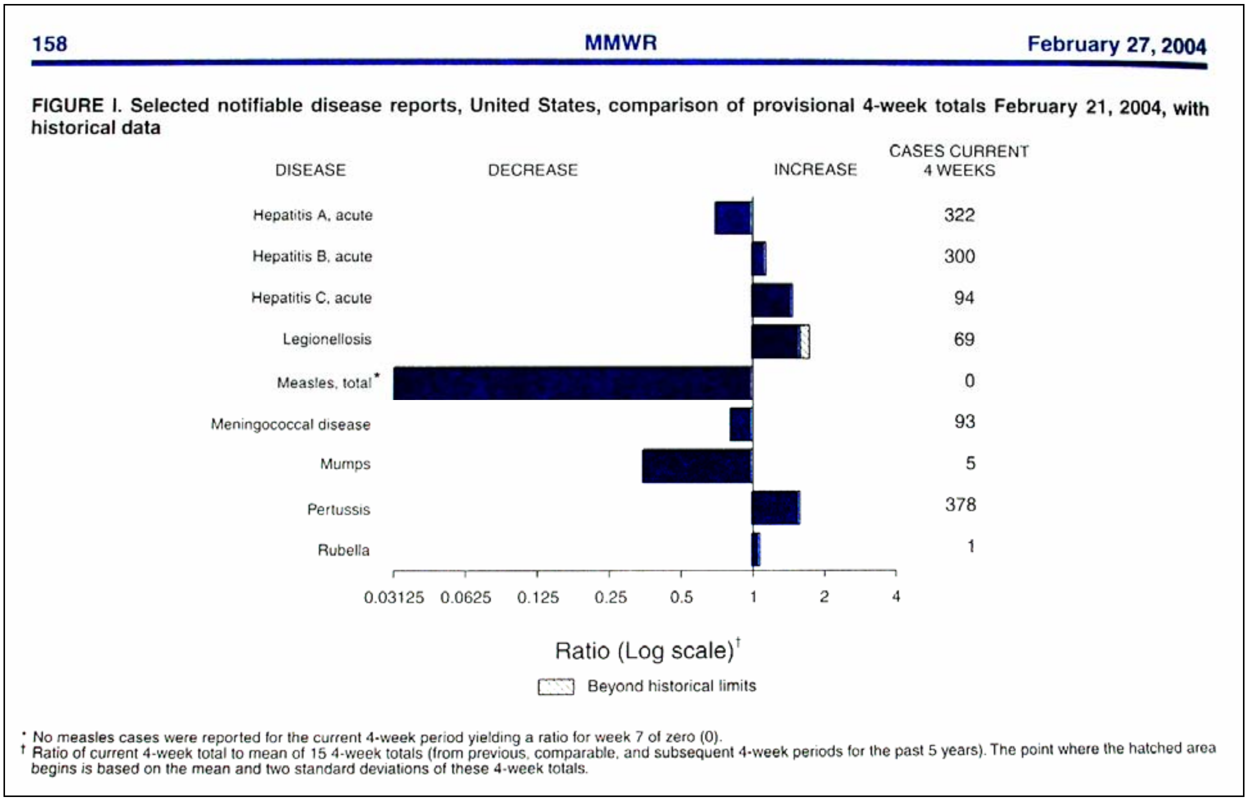


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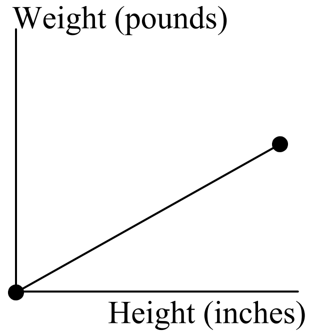
FIGURE I. Selected notifiable disease reports, United States, comparison of provisional 4-week totals February 21, 2004, with historical data

* No measles cases were reported for the current 4-week period yielding a ratio for week 7 of zero (0).
† Ratio of current 4-week total to mean of 15 4-week totals (from previous, comparable, and subsequent 4-week periods for the past 5 years). The point where the hatched area begins is based on the mean and two standard deviations of these 4-week totals.

Scalar vs. vector coordinates (points vs. lines)

Sometimes, a single number is all you need to know. For example, a single cardiac enzyme level can be all the information necessary to rule in myocardial infarct as a diagnosis. Other times, two numbers (bivariate data, to use math jargon) are needed to make sense of the meaning. For example, weight might not be particularly meaningful unless height also is given. In math jargon, a ***scalar*** is a quantity possessing only magnitude while a ***vector*** is a complex quantity possessing both magnitude and direction. You might find single numbers referred to as scalar and pairs of numbers (bivariate data) as vector quantities in some math books.

Graphs are particularly powerful ways of showing relationships, and their construction is described in more detail later. At this point, you should realize that in a two-dimensional graph (e.g. height measurements plotted on one graph dimension versus weight plotted on a second), it takes a pair of two numbers to describe each point (each point representing the height and weight of a single individual). Drawing a line between the graph origin (where the two axes meet at 0,0 units) and points representing paired height and weight produces a vector showing magnitude (distance away) as well as direction (angled up and to the right, since as height increases weight tends to increase too.). Here is an example of that line (without actual numbers) for an individual with a certain height and weight:



Working with counts vs. proportions

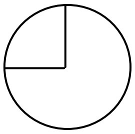
Sometimes a count is all it takes to deliver clear messages. Counting the number of “wrong limb” surgeries is sufficient since that error should never occur. Counting the number of children with leukemia who died after receiving the same wrong medication by the same wrong route year after year delivers a clear message because that invariably fatal intrathecal administration error should never have occurred anywhere again. We don’t need to calculate what percentage of surgical instruments have been accounted for at the end of each case because none are supposed to be left inside any patient – instrument counts suffice. However, in other circumstances a raw count is not meaningful. If more people die in one country than another every year, does that mean risk of dying differs between those countries? Perhaps, but if one of those countries has a larger population, or an older population, then perhaps not! Similarly, risk of catheter-associated infections in different facilities cannot be compared meaningfully by just counting infections (while disregarding the number of patients exposed to risk and their durations of risk exposure) Sometimes proportions are more meaningful. This relationship between two counts can be expressed in several ways, namely as ***odds***, ***ratios*** and ***proportions***. Sometimes we need rates rather than simple counts, for example to decide whether infectious diseases in our communities remain under control; however, understanding what public health or other rates really indicate requires knowing whether they have been constructed as ratios or as proportions.

Odds vs. ratio expressions

If you do much boating, then you’ve probably been told that 5:1 (5 to 1) is the safe way to figure out how much anchor line to use for a given depth. If you like to cook, then you might know that mixing one part vinegar to three parts olive oil (thus, representing amounts of vinegar-to-oil as 1:3) is the basis for good salad dressings. Gamblers know that the odds of winning are represented by the probability of occurrence of an event to the probability of nonoccurrence. If your favorite horse has a 20% chance of winning, then there is an 80% chance of losing so the odds are 20:80 which simplifies to 1:4 (which is why a fair bet would pay a premium of four times the amount wagered if that horse won). The meaning of a colon is what makes all this shorthand similar – the colon indicates the ratio of one quantity to (not in) another. If the directions for diluting a drug specify 5:1, those directions more likely mean drawing up one volume and adding it to five volumes (rather than drawing up one volume and mixing it into four volumes to make a total of five).

A ***ratio*** is the value obtained by dividing one quantity by another. The quantity on top and bottom in the fraction representing that division have mathematical names: *numerator/denominator*. ***Ratio*** is a general term that includes ***proportions*** (in which the denominator includes the numerator) and ***odds*** (in which the denominator does not include the numerator). To convert odds into proportions, you need to do some addition (e.g. 1:3 above indicates that your salad dressing is 1/(1+3) = ¼ = 25% vinegar, not 1/3 vinegar; 1:4 indicates that your favorite horse has a 1/(1+4) = 1/5 =20% chance of winning, not a ¼ or 25% chance). To convert proportions into odds, you need to work with complements.

Complements & reciprocals (inverses)



Compliments are the nice things we say to each other, while ***complements*** are "either of two parts or things needed to complete the whole." The quantity or amount that "completes anything" has meaning to artists (for whom complementary colors are the colors which when mixed in equal proportions produce white) and to people solving problems with mathematics. If our great salad dressing is 25% vinegar, then the other part must be 75% of the whole volume. 75% is the complement in this example, and the key to converting these proportions into an odds expression. A ratio of 25% to 75% simplifies as 1:3 (which is where this recipe started in the paragraphs above). Similarly, we could be talking about the complementary proportions of medical and surgical patients comprising the total patient population of a medical-surgical ward, the complementary numbers of patients in an experimental treatment trial who survived or died, the distribution of different health care occupations represented among a group of hospital workers, etc. In the same manner as the salad dressing example, complementary proportions can be turned into odds expressions by subtracting one portion given from the whole (the whole being represented as 1.0 in proportional or 100% in percentage notation, minus the proportion or percentage given) and using that result in a ratio.

Reciprocals (also sometimes called ***inverses*** but inverse has more than one meaning) are, by definition, just the number one divided by whatever we've got. If x represents a variable, then 1/x represents its reciprocal. A reciprocal, in mathematics, is "that by which a given quantity is multiplied to produce unity." Clearly, x multiplied by 1/x equals 1. An inverse is, more generally, "opposite to in nature or effect" or "reversed in position" (thus, addition can be considered the inverse of subtraction as a mathematical operation; a reciprocal is an inverse; in matrix algebra, one refers to an inverse matrix rather than a reciprocal matrix; on linear graphs, an inverse function is one that crosses another function at right angles; and statistical software provides various probability functions as well as inverse probability functions based on whether probability is the input or output provided). Some reciprocals are particularly useful. For example, if a particular therapy benefits 10% of those who receive it then the Number Needed to Treat (NNT, in evidence-based medicine jargon) is 100%/10% = 10 (if you prefer to work in percentage notation) and also 1/0.1 = 10 (if you prefer to work in proportional notation), which means that on the average we'd need to treat 10 people in order to expect to see one person improve. Similarly, if 5% of people who receive a treatment experience adverse effects, the Number Needed to Harm (NNH) would be 1/0.05 = 20 (for every 20 people treated, we can expect to see at least one harmed). Another useful application of inverses is to transform the number of times devices fail into the average time between failures. For example, if a brand of cheap computers crashed on the average 4 times per month, then the average time between failures would be ¼ of a month or about once a week.

Summing complementary proportions

In 1991, one preventable cause of death accounted for approximately 20% of deaths in Canadian men and approximately 20% of deaths in Canadian women. Approximately what percentage of deaths among all Canadians could be attributed to that cause in 1991? If you answered 40%, then you made the same mistake that about 30% of UBC nursing students have made on a similar exam question over the past several years. This type of problem cannot be answered correctly by simply adding the percentages!

If you made that error, think about this problem along the lines of the following experiment. Fill a one-cup measure half full. Fill a second one-cup measure half full. Next, pour both into a two-cup measure. Is the two-cup measure 50% full or 100% full?

The two one-cup measures represent complementary portions of the whole (two-cup) volume, just as the men and women represent complementary parts of the whole Canadian population. The percentage of each cup that is full (or, similarly, percentage of each gender's deaths attributable to that one cause) therefore represents a fraction of a fraction in terms of describing the entire population. This brings us to simple averages and weighted averages.

Concept of the weighted-average

Instead of adding two ***complementary proportions***, you need to compute a ***weighted average*** to solve problems like this one. For example, if the Canadian population were 60% females and 40% males, then that one preventable cause of death would account for (20%)(60%)+(20%)(40%) = 12% + 8% = 20%.

Since the same proportion (20%) applies to both men and women (all components of the total population), the answer will be simply 20% regardless of the "weights" applied - we could have got the same result just by a simple average of the two population components: (20% + 20%)/2. However, if a different proportion applied to men than to women, then a simple average would no longer deliver the correct answer! A simple average adds all the individual values and divides that sum by the number of values. A weighted average doesn't give each value equal weight. For example, if the proportion of deaths were, instead, 15% in women and 25% in men, then a simple average would yield

(15%+25%)/2 = 20% but the weighted average would yield (15%)(60%)+(25%)(40%) = 19% because it gives greater weight to the larger component (here, women).

Practice Problems

5. Infection rates in hospitals, nursing homes, and other health care institutions historically have been computed using a variety of denominators. If the number of infected patients in a given month is divided by the number of admissions or discharges during that month, is this a ratio or a proportion?

6. If instructions for reconstituting a drug indicate a 1:2 dilution, and if two volumes of that drug had been prepared as one part concentrated drug in one part diluent, then by what factor is the preparation too strong?

7. If 90% of the students in your class are of one gender and you want to compute a weighted average for a gender-associated attribute (e.g. height), how would you set up the calculation?

2. Representing Quantities as Algebraic Entities

In the previous section, as in much of our daily life, quantities are represented by numbers. Using numbers permits us to describe the score of an individual student on a particular test, the weight of a certain individual at one point in time, the price of an item that looks attractive for dinner tonight, etc. However, using numbers alone to express quantities that vary from individual to individual, place to place, or time to time doesn't permit us to explore relationships or to project estimates. For that, we need to add symbols to our mathematical vocabulary.

Constants, variables, functions, equations

Constants and ***variables*** are like words, ***functions*** are like phrases, and ***equations*** are like sentences in the language of mathematics. ***Constants*** are simply that – constant values that do not change. Some are so fundamental to science's understanding of how the world works that *algebra* gives them their own symbol. Earth keeps us all attracted to its surface by a gravitational constant, a force that doesn't change. The ratio of any circle's circumference to its diameter is a constant value (which you may remember from school as pi; π always has been and always will be 3.141592…). In a previous section on logarithms, base *e* was mentioned; it always has been and always will be 2.7182818…

Variables do change from individual observation to individual observation, so in algebra are simply represented as a letter. The letter "x" often is used to represent the unknown quantity, but any Latin letter is fine. The letter represents the concept of what is being measured, and subscripts are used to indicate individual observations (thus, if x represents test score or weight or whatever, then x_1, x_2, x_3 could represent three different test scores, weights, etc.).

A ***function*** indicates an operation or operations to be performed on an appropriate set of values. It is like a relationship rule that transforms each number x_i (the subscript i is just an index indicating there is more than one value of x, and when it is replaced by a numeral then we know which in a series of values is being specified). A function of x, often written as $f_{(x)}$, produces a single number for each value of x such that no two different $f_{(x)}$ values arise from the same x-value. A function is not the same thing as an equation, it is more of a generalization of the equation concept. If you've ever used a pocket calculator, then think about the fact that it contains function keys (you can transform any number into its inverse, logarithm, square, square root, etc. just by pushing a button – the resulting quantity is transformed from one numeric value to another).

An ***equation*** is like a mathematical sentence that says two expressions are equal. An equation must have two expressions, one on each side and both joined by an "equals" sign between them. A function can, but doesn't have to join two expressions (it can be like a phrase or incomplete sentence). An instruction to multiply any number by 9/5 or to add 32 is a function; the equation $F=32 + 9C/5$ incorporates those functions into an equation (which says temperature in degrees Fahrenheit is equal to the transformed Celsius value).

Algebra provides a language to represent constants, variables, and operations on them. Learning algebraic expression enables people to state general relationships between quantities, use a relationship formula to solve specific instances of general classes of problems, and strengthen abstract conceptual thinking.

Symbols representing variables & notable constants

As indicated above, Latin letters are used to represent variables and variables are simply attributes that vary in value from observed individual value to observed individual value. For example, the age, height, or grade point average today of every student in your class are variables. Similarly, the temperature, pulse and respiration rate of a patient at different points in time are variables. Rather than write out the entire name of each variable, we can abbreviate by using a Latin letter (e.g. a for age, h for height, g for grade point average, t for temperature, p for pulse, r for respiration rate, or any other letter we want). Some people prefer to use just two letters, x and y, for all *independent* (x) and *dependent* (y) variables in a problem.

The grammar of algebraic notation holds a few more rules about using letters, and a few embellishments that can be added to letters to communicate meaning even more clearly. You've already read about using a lower case letter and a subscript. Lower case letters are used to indicate a variable observed in a sample from a population, upper case is used to indicate that same attribute observed in the entire population from which samples are drawn (to be precise: ***variables*** occur in ***samples***, ***parameters*** occur in ***populations***), and subscript numbers to indicate specific observations of that attribute among the "n" number of observations possible. The letter N is used to represent ***population size***; the letter n is used to represent ***sample size***. Continuing with the example of a patient's temperature recorded four times each day, that attribute could be represented as t_{ij} where "t" represents the variable temperature, subscript "i" represents individual days of hospitalization, and "j" represents which of the four daily recordings (so, for example, $t_{3,4}$ would be algebraic short-hand for the 4th temperature recorded on the 3rd day). If you're one of those people who wants to use just x or y to represent all variables, then in a *multivariate* problem you'll have to use subscripts to indicate which of the variables is meant (e.g. x_1 might refer to age, x_2 might refer to height) and double-subscripting to indicate which individual observation (e.g. $x_{1,1}$ for age of individual #1, $x_{1,2}$ for age of individual #2, $x_{2,1}$ for height of individual #1, and so forth).

The appearance of letters described thus far indicated an ***observed*** value; however, algebra also provides a way of indicating ***predicted*** and ***averaged*** values. For example, since chocolate and espresso are essential parts of my healthy diet, the amount of chocolate content (which I'll call variable c) and espresso intake (variable e) per day might be measured. Based on these measurements, the chocolate content of a good week could be estimated (predicted) as a function of some other factor and the algebraic way to indicate a predicted value is to put a "hat" above a variable's letter (thus $\hat{c}$). Similarly, the espresso intake over a week could be averaged, and the algebraic way to indicated averaged value is to put a bar above the variable's letter (thus $\bar{e}$). Thus, in any function or equation that contains variables c and e it would be possible to indicate clearly whether reference is being made to an observed value (c or e), a predicted value (c-hat or e-hat), or an average value (c-bar or e-bar).

A few Latin letters tend to be reserved for special meaning. As indicated above, N and n frequently are used to indicate population and sample size, respectively. Lower-case subscript letters i and j frequently are used as index counters (to specify a particular observed value). Also as indicated above, the letter *e* appears in mathematical expressions to represent a long number that is the base of "natural" or Napierian logarithms (to 7 decimal places, *e* is 2.7182818, and its precise value keeps going many more decimal places beyond that!). Other constants and statistical values tend to be represented by Greek letters like alpha, beta, lambda, mu, pi, sigma, etc. (α, β, λ, μ, π, σ, etc.).

To summarize thus far, symbols representing variables and constants are like nouns in a mathematical vocabulary. Accent marks on those symbols indicate whether a variable is the observed, predicted, or average value. In order to do something with these quantities, verbs are needed and for this we have symbols representing operations.

Symbols representing operations

Addition and subtraction, multiplication and division are the fundamental things we can do to quantities whether those quantities are represented by numbers or by algebraic symbols. There are several ways to indicate each of these operations.

Addition and subtraction can be indicated by familiar plus and minus symbols (as in a+b and a-b), but if there is a large number of observations to add then it would be too cumbersome to have to put in all those + marks between all those values. A shorthand way of indicating that a series of numbers is to be added is to use the Greek letter sigma (as in $\mathbf{\Sigma x_i}$ which means to add up all the individual values of x_i, and if only certain values are to be added then expressions like ${}_{i=1}^{5}\Sigma x_i$ indicate that just observations 1-5 should be added).

Multiplication can be indicated by the familiar "x" symbol (as in 5x5=25). However, in algebraic notation an "x" between values might be confused with an "x" used to indicate a variable, so it is more common to see (x)(y) or $x \cdot y$ or even just xy as an indication that variable x is to be multiplied by variable y. Just as there was a Greek letter symbol to indicate a large series of numbers is to be added, there also is a Greek letter symbol to indicate that a large series of numbers is to be multiplied. **Π** (the upper-case version of the letter π) is used (as in Πx_i to indicate that all the individual values of x_i should be multiplied by each other).

Division generally is indicated by placing one quantity above a / and the other quantity below that slash. Since division by a number is the same thing as multiplying by the reciprocal (inverse) of that number, dividing x/y also could be indicated by xy^{-1} since y^{-1} means 1/y. That last example deserves special attention: xy^{-1} is not the same thing as $(xy)^{-1}$. If the distinction isn't clear now, it will become clear when the use of parentheses is explained in the Order of Operations section below.

Multiplying something by itself two or more times has a special notation. In the y^{-1} example above, the superscript –1 is called an ***exponent***. Multiplying a number by itself twice is indicated by the exponent 2, by itself three times by the exponent 3, and so forth (as in $2^2 = 4$ and, as you might guess, $2^{-2} = ¼$). A related operation, roots of numbers, is indicated by using a fraction as the exponent (the cube root of 8 is 2, the square root of 4 is 2, so $8^{1/3} = 2$ and $4^{1/2} = 2$). The symbol √ which you might remember from math classes way back in grade school as the symbol for principal (non-negative) square root is also but less commonly used to indicate taking a principal (non-negative) root.

Practice Problems:

1. The function (n+1)/2 returns the middle value (which is called the median) where "n" numbers are arranged in ascending order. What is the median in a series of 9 numbers? 100 numbers? 101 numbers? 1000 numbers?

2. In the exponential notation 2^x, if x is a positive number then the product is a positive number and if x is a negative number then the product still is a positive number. What value does 2^x take if x=0?

3. In a prior section, the concept of simple and weighted averages was introduced. The simple average, also known as the mean, can be written in algebraic notation as $\Sigma x_i/n$. If the weight applied to each observed value is w_i, then how would a weighted average be written in similar notation?

4. What is the product from multiplying $(x^3)(x^2+3)$?

Concept of ═ vs. ≡ (equivalent vs. identical); inequalities

Equations and some of the expressions you've seen so far use the symbol = to show that whatever is on the left side of that symbol is equivalent to what is on the right side. Just as two five-dollar bills are equivalent to ten dollars but not identical to one ten-dollar bill. Since two sides of an equation are equivalent, both sides can be manipulated in ways that keep both sides equivalent (so dividing $10 by 2 isn't like tearing a ten-dollar bill in half). That also can lead to solutions that have no physical meaning (e.g. solving $x^2 = 4$ for x leads to the mathematically valid solutions of $x = 2$ and $x = -2$, but negative numbers might not make any sense if x represents a pulse or respiration rate). Similarly, solving $x^2 – 2 = 0$ or $x^2 + 2 = 0$ could lead to solutions of $x = \pm\sqrt{2}$, which is a real number, and $x = \pm\sqrt{-2}$, which is not a real number (since it is not possible to express the square root of a negative number). The symbol ± indicates that the value following may be positive or negative.

Inequality statements are used to limit a solution to a specific ***interval***. Instead of the = symbol linking two sides of an equation, inequality statements use $\neq$, <, $\leq$, $\geq$, or > to indicate that the left side is not equal, less than, less than or equal, greater than or equal, or greater than the right side.

Practice Problems:

5. What does the inequality statement $0.05<p<0.10$ say about the variable p?

6. If the function of x is defined as x^{-1} then there is a set of values that the variable x can take and a set of values that the function of x can produce. These are called the domain and range of the function. For $f_{(x)}=x^{-1}$ what is the domain of the function? (hint: what single value can x not take)

7. Quetelet's index is used as an index of body mass in physiology and anthropometrics. His index is defined as weight in kilograms divided by square of height in meters. Write this as an algebraic equation. Given that one kilogram equals 2.2 pounds and one meter equals 39.4 inches, what is an equivalent equation based on pounds and inches?

3. Numeric & Algebraic Operations

Our mathematical vocabulary at this point contains the equivalent of nouns (represented by numbers and symbols for constants and variables) as well as verbs (represented by symbols for addition, subtraction, multiplication, division, square roots, and exponential powers). Punctuation comes next – the mathematical version of rules of grammar and symbols analogous to things like commas, semi-colons and periods that indicate groupings and places to pause.

Order of Operations

"BODMAS" (or acronyms made up from those letters as a memory aid) stands for Brackets, Ordinals, Division, Multiplication, Addition, Subtraction. Some students may be more familiar with "BEDMAS" (same idea, with the word Exponents substituted for Ordinals). That is the order of precedence in which a sequence of mathematical symbols is processed, essentially a simple rule of grammar. Just as there is a world of difference in meaning between "What is this thing called love?" and "What is this thing called, love?" there is a world of difference between $5+2\cdot3$ and $(5+2)(3)$. BODMAS instructs that $5+2\cdot3$ means multiply $2\cdot3$ and then add 5 to get 11, versus add $(5+2)$ and then multiply by 3 to get 21. First do any operations contained in brackets (and if there are brackets within brackets, work from the inside out), then raise anything to ordinal powers indicated, then do any division or multiplication, and finally do any addition or subtraction. Ordinals can been applied to individual numbers, like 2^2, or to individual symbols, like x^2, or to groupings within brackets, like $(x + 2)^2$.

Parentheses in math therefore are like commas in prose – they mark the beginning and end of groupings. Parentheses are one form of brackets; you've probably seen [] and {} as well as () and all three bracket styles can be used to make nested bracketing more obvious. $\{5[(3+2)/6] – [(3)(2)]\}$ is, for example, a bit easier to decipher than the same expression written in just one bracket style like $(5((3+2)/6) – ((3)(2)))$. Both are correct, but the first style is easier to navigate when expressions become complicated.

The equal sign and inequality symbols ($=, <, \leq, \geq, >$) are like semicolons, colons or periods. They separate the mathematical equivalent of sentences. Unlike semicolons, colons or periods, they also indicate the relationship between two adjacent expressions.

Another item of punctuation common to both text and math is the ellipsis (which is the name of those three periods you see indicating omission from a sentence of a word or words that would complete or clarify…). In algebra, an ellipsis positioned in a function like $(n)(n-1)(n-2)(n-3)\ldots(1)$ indicates that the series should be completed (in this case, multiply n by n-minus-1 by n-minus-two and so on until the series finally reaches multiplication by 1). The ellipsis does not alter order of operations, it simply permits shortening of expressions.

Practice Problems

1. If x=2 and y=4, then what is the value of the following expressions:
    a. $[(x+y)^x]^{1/x}$
    b. $xy/x + (-x)(-y)$
    c. x^{-3}

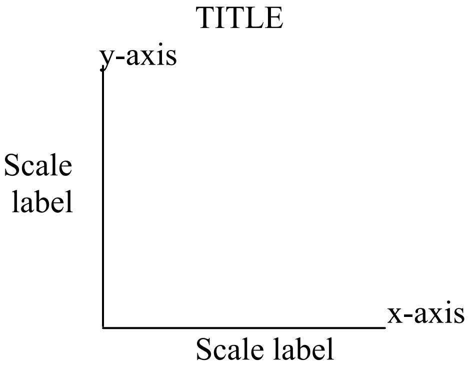
2. Express the following instructions in algebraic notation:
    a. Multiply x times the sum of x and y, then divide by y.
    b. Add every integer smaller than y to the integer y until –3 is passed.
    c. Two times a number is the same as that number squared.

Graph Construction

Up to this point, the concentration has been on numeric methods. It has been observed that numeric methods are data reductive, condensing rich data sets down to just a couple of single numbers as summary statistics. Since most of the tools developed for data analysis, for rigorous examination to discern trends in data sets, have been numeric methods, numeric methods have been the mainstay. However, graphical methods preserve the richness of data sets. They "allow us to explore data to see overall patterns and to see detailed behavior; no other approach can compete in revealing the structure of data so thoroughly." This quotation is from one of two surprisingly inexpensive books strongly recommended for adding to your library if you plan to do much work with data analysis or graphs. They are *Visualizing Data* (1993) and *The Elements of Graphing Data* (2nd edition, 1994), both by William Cleveland (a researcher at AT&T who devoted his career to studying visual coding and decoding of information) and published by Hobart Press of Summit, New Jersey. These two books explain fundamental principles of graph construction, and scientific findings on visual perception, as well as provide schemes for graphical analysis of progressively more complex data types. Another book that should be in the library of anyone serious about creating graphics that clearly portray data without introducing optical illusions that plague common business-oriented graphics is Edward Tufte's *The Visual Display of Quantitative Information* (1983, Graphics Press, Cheshire, Connecticut).

This brief section cannot convey the level of understanding you can attain from studying the books by Cleveland and Tufte. As well, there are many types of graphs and it is not the intent here to teach usage of every type. Instead, the learning objective for this brief section is to review basic terminology and rules for graph construction.

At its most basic level, component parts of a graph and the name of those parts are:



More complex forms exist, such as 3-dimensional graphs and polar coordinate plots and more, but this simple frame illustrates the basics. A ***title*** should briefly but clearly tell readers what is portrayed; it may appear immediately above or below the graph. If necessary, a caption, legend or footnote beneath the title can be used to tell additional detail (such as source of information, type of adjustments applied to the data, definition of plotting symbols, etc.). ***Scale labels*** should indicate what units pertain to reference values aligned in progressive order along the axes. Typically, there is an ***independent variable*** and a ***dependent variable*** (essentially, a predictive value and an associated response value); independent variables traditionally are plotted on the x-axis and dependent variables on the y-axis. For example, one might indicate progressive calendar years on the x-axis and corresponding rates of disease on the y-axis to study trends in disease rates. An axis scale label would clarify units, such as “Cases per 100,000 of population” or “Total number of cases reported”, and offer just enough equally-spaced reference values, tick marks, or grid lines along the axis to facilitate interpretation of plotted data point location. It seems a bit pedantic, but you also may see the x-axis called the abscissa and y-axis called the ordinate in textbooks or software that use these more formal Cartesian coordinate names.

Individual data points are plotted within these horizontal (x-axis) and vertical (y-axis) scales, using plotting symbols (e.g. dots, circles, triangles, etc.) that may be joined by a line if desired. Each point represents paired “bivariate” data values, and the constellation of points reveals trends within relationships between the two variables (e.g. a rate of disease might be increasing, decreasing, or stable over various periods of time). Connecting lines, which can be drawn to connect each point in a jagged manner or drawn smoothed to come as close to all points as possible without being jagged, may be added to make those trends more obvious. In order to make visual decoding of this information as easy and accurate as possible, there are several rules to consider.

First, keep graphs as unembellished as possible so that data rather than “chartjunk” (superfluous elements) stand out. Avoid using too many tick marks on axes or frame lines or complex plotting symbols, which just clutter the image. Select plotting symbols that are prominent, therefore easily seen, and take special care when plotting symbols must overlap. Be careful when plotting more than one set of values on one graph. Sometimes more is gained by simplifying an overly complex graph, moving keys or labels outside the rectangle or dividing very complex pictures into several graphs. There are other aspects to refine clean, clear presentation of visual information; they are more technical and better explained in the books cited above.

Second, reinforce clear understanding of information portrayed in a graph. Captions should describe everything that is graphed, draw attention to important features, and describe conclusions drawn. “Error bars” drawn through each plotting point to convey information about variability should be explained (make clear what they indicate, be it range, standard deviation, standard error, or confidence interval). If data values are expressed in logarithmic form (as often is done to more effectively portray rate of change rather than absolute amount of change between data points), be sure that scale labels indicate this. Plot with a 0,0 origin and make every possible effort to avoid scales that contain breaks (which tend to inflate impressions of the magnitude of value changes). Again, there are additional technical considerations (considerations like aspect ratios and banking) beyond the scope of this introductory review – see the books cited if you’re interested in studying this further.

Third, try to select the most effective format for your particular problem. The right format permits large amounts of information to be graphed in relatively small amounts of space without becoming too cluttered or convoluted - far more efficiently than trying to say as much in a table or paragraph. Treat graphing as you do writing – iteratively experiment and edit to discover the most effective presentation; see if presenting two or more graphs of complex data promotes more effective comparisons than a single picture. Again, to gain better understanding of which graph types better suit which data problems, see books referenced above and plan on life-long learning through reflective experience.

One last piece of advice: Be methodical and cautious in interpreting what you see in graphical presentations. Human minds are very sharp at recognizing visual patterns, but we also tend to be less critical when interpreting pictures as opposed to text. In their book *Making Sense of Data* (Abramson JH, Abramson ZH. 3rd edition, 2001, Oxford University Press), the authors advocate an excellent three-step process. First, summarize the facts to say what is known without doubt (at this stage, do not make any assumptions or guesses or interpretations – just summarize the facts!). Next, after recognizing exactly what is shown, list all possible explanations for relationships perceived among the variables (some explanations may be more plausible than others, and many may depend on certain assumptions). Third, having identified the facts, assumptions and possible explanations for any patterns perceived, ask yourself what else if anything else must be known to make an interpretation. If you have sufficient information, make a decision based on accepting the most plausible explanation; if not, frame questions for additional research that must be addressed before you can accept a specific interpretation.

Simplifying & solving equations in 1 and in 2 variables

Back to high school algebra! Since many health events are multifactorial, health care professionals will encounter equations more complex than those presented thus far in this review. In order to summarize interrelationships or predict outcomes with the sort of mathematical models often produced by regression analysis, it is necessary to consider equations containing more than one independent variable. Software used for workload management (e.g. Omega or PRN scoring systems) or prognostic classification of cases (e.g. APACHE) incorporates many predictor variables. This section briefly reviews fundamental aspects of solving simple and more complex equations.

The graph of a simple equation containing just one dependent and one independent variable shows many possible solutions along a straight line if the variables are all first-order (not raised to higher powers by an exponent, e.g. $y=2x$). If the independent variable is raised to a higher power (e.g. $y=x^2$), then the graph will show a curved line. For example, growth charts show typical heights (the dependent variable) for given ages (the independent variable), so one can look up expected value for any child based on their age. Lines on charts like these follow a mathematical equation, so the same value can be estimated either by solving an equation for a specific value of x or by looking on the graph for the y value corresponding to a given x value. Again, x represents the independent (or predictor) variable and y represents the dependent (or response) variable.

So if we can look up values on charts, or let computers automate all the number crunching, why bother learning algebra? In health care settings, we're often faced with reams of information presented as numbers along with questions posed vaguely in words. Learning algebra as a way to express variables and their relationships is a useful skill because its discipline helps sort through the extraneous stuff, identify which variables are of interest, turn vague questions into specific hypotheses, better understand relationships, and make more accurate predictions. Charts might not always be available for solutions we need, and computer-generated answers have been known to be wrong (sometimes so wrong that they can injure or kill! - Birnbaum D, Morris R. Artificial Stupidity. CLINICAL PERFORMANCE AND QUALITY HEALTH CARE. 1996;4(4):195-7.).

Another reason to learn algebra is to tease out details that exist "between the lines" in journal or conference reports. Space is limited, especially in abstracts or posters, so what is written might not be what you really want to know. Sometimes, for example, authors are coy about stating actual costs or doses or response rates – they express those values in relative rather than absolute terms. As a hypothetical example, a report might read that two drug regimens were compared, response rate with one regimen was three times as fast as the other and difference in response times was 30 minutes. That's fine if all you want to know is how those two drugs compare against each other, but what if you want to know how fast the response time was for both of them compared to yet another drug currently used by your own patients? Try using algebra now (before you read the next paragraph!) to discover what the actual response times were for the two drugs in this example.

You might see that there are two variables in this problem, response time for one drug and response time for a second drug. That's fine for now, but to be technical one of the variables is drug (with two levels, drug #1 & drug #2) and the other variable is response time - this technical distinction won't make any difference for the algebra practiced here, but will be important if you want to use sophisticated computer software to solve problems. You also should see that we've been told two things about the relationship between them: one time is thrice the other and difference between their response times is 30 minutes. Variables can be named anything you like; I'll call mine D_1 and D_2 here, with D_1 arbitrarily chosen to represent the drug with the faster response time. The first of the two relationships can be expressed as $D_2=3D_1$ (or $D_1= D_2/3$ if you prefer) and the second relationship as $D_2-D_1=30$. Since an = symbol means that everything on its left is equivalent to everything on its right, equations can be manipulated so long as we do the same thing on both sides to maintain that equivalency. By adding D_1 to both sides of the second equation, it can be "solved for D_2" in the sense that like terms have been gathered together and the variable "solved for" is isolated on one side of the equation. Thus, the original equation $D_2-D_1=30$ can be transformed into $D_2 =30+D_1$ by adding D_1 to both sides. We now have two equations both "solved for" D_2 since $3D_1$ and $30+D_1$ both equal D_2. Since they're both equal to D_2 we can set those two functions on either side of an = symbol and at last solve for the actual value of D_1 (what we wanted to know in the first place, response time for the faster drug). Again, doing the same thing to both sides of an equation to gather like terms and isolate the variable of interest on one side, $3D_1 = 30+D_1$ can be transformed as $3D_1-D_1 = 30+D_1-D_1$ or $2D_1=30$. Thus, the faster drug had a response time of 15 minutes and we can obtain response time for the slower drug (45 minutes) from either $D_2=3D_1$ or $D_2 =30+D_1$. The times of 15 and 45 minutes for the two drugs are the only values that satisfy both relationships expressed in our equations.

Equations like these are called ***first-order or linear equations*** because there are no variables raised to higher powers and their graph is a straight line. Equations with a squared term (e.g. x^2) are called ***second-order or quadratic*** (and their graph is a curve, not a straight line); those with cubed terms (e.g. x^3) are called ***third-order or cubic*** equations (and they graph as complex curves containing ***inflection points*** – points where curvature changes from convex to concave or vice versa). Solving higher-order equations is beyond the scope of this simple review, but the method follows similar lines: Gathering like terms, reducing the variable of interest to first order by factoring or taking roots, and then solving for unique numeric values that satisfy the equation.

First-order or higher-order equations containing more than one dependent or more than one independent variable are called ***multivariate*** equations. When equations contain more than one independent variable, then a range of dependent variable values exist for any given value of any one independent variable. For example, a complex staffing model might take into account the care requirements of patients based on case complexity, the nature and amount of work that can be done by different types of workers (e.g. RNs vs. LPNs vs. care aides, or physicians vs. physician-assistants or nurse-practitioners), as well as the cost per hour for each type of worker, so that a system of equations defines optimal staffing mixes. Similarly, multi-compartment pharmacology equations might take into account body mass and renal function as well as half-life of a drug in order to define optimal dosaging. Since many health and health care related events are multifactorial, multivariate equations are not uncommon in modeling or analyzing these events. However, multivariate methods also are beyond the scope of this review. If, now or later, you need a gentle introduction to principles underlying multivariate methods, a book worth examining is Flury & Riedwyl's *Multivariate Statistics, A Practical Approach* (Chapman & Hall, London, 1988).

Combinations & permutations solved with factorials

Arrange a quarter, a dime, and a nickel in one row on a table. How many different ways can the three coins be rearranged? There are three choices for the first position, two coins left for the second position choice, and the left-over coin comes third: Thus, there are (3)(2)(1)=6 different ways to arrange three coins in a row. To be technical, this is called a ***permutation*** (how many ways can a series of items be ordered) and the related multiplication of successively diminishing numbers is called a ***factorial***. A shorthand way of saying "multiply 3 by every smaller integer" is to write 3! (which reads 3 factorial). By definition, 0!=1 and that's where the multiplication sequence of smaller integers ends. Small factorials can be computed by hand, and most calculators have an n! key (beware, though, factorials easily produce numbers too large for calculator memory).

In the experiment above, each arrangement was considered different because the coins appear in a different order despite the fact that all six arrangements contain the same coins. If order is important, we're talking about permutations; if order isn't important (if we're only concerned whether each arrangement contains different elements), we're talking about ***combinations***. With those three coins arranged three at a time, there are 6 permutations but just 1 combination.

Try the experiment again, but this time select two of the three coins each time and decide how many different arrangements of 2 can be made as well as how many of those arrangements include the quarter. Again, there are 6 permutations possible, but this time there are 3 combinations in total with 2 in which a quarter is one of the two coins in an arrangement.

Since these kinds of calculations are at the heart of probability estimations, fortunately there is a computing formula for permutations and another for combinations (a much better alternative than counting the possibilities on our fingers and toes!). If you study statistics, you'll learn these formulae.

Round-off error

Every multiplication or division in a chain of calculations introduces the prospect of compounding a mistake – "round-off" errors that accumulate. If precision of answers is reduced by rounding off results used in subsequent calculations, imprecision grows with the number of calculations performed. This leads to two general rules: perform as few calculations as possible, and avoid rounding off preliminary results prematurely. When you study statistics, for example, you'll find special calculating formulae used for quantities like variance & standard deviation in place of formulae based on the formal definition of these quantities. The reason is that these calculating formulae involve fewer multiplications and divisions, so less potential distortion from round-off error. If calculations are performed on a computer or a good pocket calculator, their 10-15 place accuracy helps produce a more accurate end result than if calculations are done by hand with relatively few decimal places maintained at each step. **Don't round off your**

answers until the final step in any chain of calculations, and **carry at least two more decimal places throughout preliminary calculations than you want to show in your final answer!**

Practice Problems:

3. A parent is twice the age of their child, and the difference between their age in years is 30. How old are the parent and child?

4. What would a graph of $f_{(x)}=x^2$ look like if x took integer values from –5 to +5?

5. There are 15 patient beds in your ward, and constant complaints about which patients are assigned to which bed. As a discussion aid, you're thinking about making up a chart to show all the different ways the current 15 patients could be arranged. If it would take five minutes to complete a picture of each arrangement, how long would it take to make a chart showing all possible arrangements?

4. Math in Nursing & Public Health Practice

The purpose of this section is not to teach how dosage, drip rate, disease occurrence rates, etc. are calculated. Rather, the purpose is to identify good math habits that will help you avoid errors commonly made by practicing health care professionals. To safeguard your calculations, consider adopting the following habits.

First, always label numbers with the units they represent. Rather than perform countless calculations and then add a label to the answer, place labels adjacent to every number to confirm their product makes sense. If, for example, drug concentration is expressed in milligrams and dosing instructions specify 25 mg/kg of body weight, something clearly is wrong in a calculation of (25 mg/kg)(97 pounds) instead of (25 mg/kg)(44.1 kg). The first units produced (mg pounds/kg) simply don't make sense, signaling that the product itself (2425) likely is a wrong number. Similarly, with a conversion factor of 2.2 pounds per kilogram, error converting weight from pounds to kilograms is more likely to be noticed in (2.2 lbs/kg)(97 lbs) than in (2.2)(97). The former produces an answer with the label of pounds-squared per kilogram, which simply doesn't make sense. Instead, (97 lbs)/(2.2 lbs/kg) is needed to produce an answer in kilograms.

Second, be consistent in number of significant digits shown to the right of the decimal point, and always show a numeral (even if 0) to the left of decimal points. It is more difficult to mistake 20 and 2.0 or 2 and .2 for each other when consistently written as 20.0, 2.0 and 0.2. Neatness and alignment matter! In that context, I don't know if 1 and 7 are more often mistaken for each other in North America than in Europe (since Europeans tend to cross their sevens), but influence of distinctive numeral shape would be interesting to consider.

Third, as a general rule, carry at least two more digits to the right of the decimal point throughout all preliminary calculations than you want in your final answer. Avoid rounding off to the final precision wanted until the very last step. This will help avoid accumulation of round-off error compounding throughout the calculations.

Finally, rather than just redo the same calculation as a double-check (perhaps therein replicating the same calculation error), estimate magnitudes for a ball-park range on the final answer. If a rough estimate of magnitudes indicates an answer should be some product in the hundreds, be suspicious if your precisely calculated result is far out of that range.

There also are rules of thumb to help safeguard interpretation of graphs and charts. As already described, first be very sure you understand what data are portrayed. Read the title, axis labels, any legends and footnotes to summarize undoubtable facts (don't make any assumptions, guesses or interpretations – just summarize the facts!). Next, after recognizing exactly what variables are shown and in what form, consider explanations or conclusions while bearing in mind any prudent concerns or limits to interpretations given the nature of data portrayal (Beware – optical illusions inherent in certain graph forms make accurate decoding of those forms extremely difficult if not impossible! Amusing examples are provided in Huff D. *How to Lie with Statistics*. Norton, New York, 1954; scientific explanations in the Cleveland books previously cited). Third, having identified the facts, assumptions and possible explanations for any patterns perceived, ask yourself what else if anything else must be known to decide on an interpretation.

Tables may be less prone to misinterpretation than the same information pictured in graphs or charts. However, tables also can be confusing. Rushing to extract numeric values before fully understanding what the table values portray is a common cause of error in using statistical distribution probability tables. Trying to add numbers presented horizontally rather than vertically is unnecessarily difficult but sometimes necessary if tables are poorly oriented. Abbreviations and exponents can be confusing because styles often are not consistent (Day, for example, notes that *Journal of Bacteriology* uses "cpm x 10^3" while *The Journal of Biological Chemistry* uses "cpm x 10^{-3}" both to indicate thousands of counts per minute. Day RA. *How to Write & Publish a Scientific Paper*, 5th edition. Oryx Press, Phoenix AZ, 1998). Consider table layout and look for adequate clarification in titles, legends, and footnotes before looking up table values. Be careful if it is necessary to convert table units from one system to another (see good math habits listed at beginning of this section). Finally, the more complex the table the more important it is to double-check your work. Inspect complex tables twice to double-check that you've navigated to the right position and extracted the correct values.

5. Questions for Numeracy Self-Assessment

Higher-level courses in analytic techniques and statistical methods that you will take during a degree program probably are quite different from high school or undergraduate arithmetic or basic statistics courses you may have taken. They focus on higher-order learning objectives (ability to apply & explain, not just identify & define), building conceptual knowledge and comprehension rather than just memorization, and you will be expected to engage in discussion rather than learn passively through lectures. In order to succeed, you need to be able to speak the language that will be used. If you can answer questions like those that follow, then your numeracy skills are strong enough to take a higher-level course now. If you can't answer at least half the questions in each section correctly, then you should take advantage of numeracy self-study materials available before undertaking such analytic techniques courses.

Notation

1. Determine the value of x in each case.
   A. $x = (25)^{1/2}$
   B. $x = \sqrt{25}$
   C. $x^2 = 25$

2. Solve $-(x - 2)(x - 6) > 0$

3. Is $(2x)^5$ equivalent to $2x^5$? Explain.

4. Given the values x=2, 4, 6, 8, 10, for i=1 to n what is the value of Σx_i?

5. Given the values x=1, 2, 3, 4, 5, for i=1 to n sketch a graph of i vs. x_i

6. Which of the following is a function and not an equation?
   A. 32
   B. $x + 32 = 0$
   C. $32x + 16y = 7$
   D. $x + 32 < 0$
   E. $x + 32$

Operations and Order of Operations

7. Solve $\sqrt{x+2} - 5 = 0$

8. Solve $\sqrt{x+2} + 5 = 0$

9. Solve $[2^3][6 - 5(12 - 4 \text{ x } 5)]/3^3$

Working with rates, ratios, proportions, odds, percentiles

10. Rosa invested \$15,000 in two funds. One was risky but paid 11% interest. The other was safer but paid only 6% interest. Rosa made \$1250 interest in the first year. How much money did she invest in each fund?
    A. \$10,000 at 11% and \$5,000 at 6%
    B. \$7,000 at 11% and \$8,000 at 6%
    C. \$12,000 at 6% and \$3,000 at 11%
    D. \$6,000 at 6% and \$9,000 at 11%
    E. None of A, B, C, and D.

11. In 1991, one preventable cause of death accounted for approximately 20% of deaths in Canadian men and approximately 20% of deaths in Canadian women. Approximately what percentage of deaths among all Canadians could be attributed to that cause in 1991?

12. If the probability of dying during a risky surgery is 80%, then the odds against survival are ??:??

13. A market survey determined that 40% of teenagers watch television news, 30% read the newspaper, and 20% watch television news and read the newspaper. What is the probability that a randomly chosen teenager does not read the newspaper or watch television news?
    A. 90%
    B. 80%
    C. 50%
    D. 20%
    E. 10%

14. The University of British Columbia is located in an earthquake region, so we often hear about numbers on the Richter Scale. It is described as a logarithmic scale because each number on that earthquake scale represents relative change in powers of 10. A magnitude 3 earthquake is how much more powerful than a magnitude 1 earthquake?
    A. 3 times as powerful
    B. 10 times as powerful
    C. 30 times as powerful
    D. 100 times as powerful.
    E. 10^3 times as powerful

Combinations, Permutations, and Factorials

15. There are five empty desks in a classroom. In how many ways can two students be assigned to these seats?
    A. 5!
    B. 2!
    C. 52
    D. ${}_5P_2$

E. $_5C_2$

16. Simplify:

$$\frac{n(n-1)(n-2)(n-3)\ldots(2)(1)}{(n-2)(n-3)\ldots(2)(1)}$$

6. Appendix: Answers to Practice Problems & Self-Assessment Questions

A. Practice Problems

Practice Problem 1-1 (Answer: +3 and –3 both indicate the same magnitude. They differ in direction, not size. To put this in more graphic terms, think about measuring the difference in our height. Would the magnitude be different if I subtract my height from yours versus yours from mine, or would it just change the sign on that magnitude?)

Practice Problem 1-2 (Answer: Integers are whole numbers, all integers are rational numbers, but rational numbers also include the mixed numbers that can be produced by dividing one integer into another – for example 3/2 or 1.5 is a rational number but not an integer.)

Practice Problem 1-3 (Answer: No. As logarithms approach 0 from a positive direction, the value represented approaches 1 because by definition any number raised to the zero power equals 1. As logarithms depart from 0 in a negative direction, the value represented continues to become smaller and smaller because, as explained in another section below, negative exponents tell us to take the inverse of a number. At negative infinity, the value represented is as close to zero as it is going to get, but still infinitesimally larger than 0. In mathematics, being able to approach but never actually reach a value is called an *asymptote*; the value 0 is an asymptotic value for logarithmic expression.)

Practice Problem 1-4 (Answer: For the various diseases shown, the number of cases reported ranges from 0 to 378. The purpose of this figure is to show how disease occurrences in the current reporting period compare with previous periods, so a ratio is a meaningful representation and a logarithmic scale shows these ratios more compactly and uniformly than raw numbers or an arithmetic scale could. Since each scale interval represents doubling, or halving, of the occurrence of any disease, it is easier to detect relative rate of change in this log-scaled format. Diseases with large numbers here (viz. hepatitis A, hepatitis B, pertussis) would dwarf diseases with small numbers of cases (viz. legionellosis) if absolute numbers were portrayed on an arithmetic scale; all large relative changes and excesses beyond historic limits (such as for legionellosis) are more readily observed with logarithmic scaling. And if the concept of ratios is new to you, rest assured that it will be reviewed in the latter part of Section 1.)

Practice Problem 1-5 (Answer: It is a ratio. The numerator, number of infected patients, is not necessarily included in the denominator, number of admissions or discharges during that time period. Think, for example, of a nursing home with 100 residents but just 5 admissions or discharges in a month – calculated in this manner, if 10 of the 100 residents became infected that would produce a 200% infection ratio! A more meaningful expression for comparisons among facilities involves calculating a proportion among total number of patients at risk.)

Practice Problem 1-6 (Answer: The instructions indicate final concentration should be 33%. The preparation has a concentration of 50%. Therefore, the preparation is too potent by a factor of 50/33 or 1.52. Another way of expressing this is (50-33)/33 = 0.52 or 52% too strong. In mathematical notation, the symbol ":" means one quantity "to" rather than "in" another.)

Practice Problem 1-7 (Answer: If 90% are of one gender, then 10% must be of the other. Finding an average value for the attribute in each of the larger and smaller complements, which we can represent as x_1 and x_2, then weighting those values appropriately, forms the basis for calculation. The larger complement should be weighted 9 times more heavily, so expressions like $(90x_1 + 10x_2)/(90+10)$ or $(9x_1 + 1x_2)/(9+1)$ all give the same answer.)

Practice Problem 2-1 (Answer: For an odd number of observed values, there can be a middle observation which has an equal number below and an equal number above; for 9 values, the median is the 5th one. For an even number of observed values, one of the observed values cannot be the median because there wouldn't be an equal number of values below and above – so the median must be the average of the middle two numbers: for 100 values, the median would be positioned at the 50½ point or the average between observation #50 and #51. The function given indicates that median for 101 values would be the 51st and for 1000 would be the average of the 500th and 501st.)

Practice Problem 2-2 (Answer: By definition, anything raised to the 0 power equals 1. For $x>0$, the product would be progressively increasing positive integers; for $x<0$, the product would be equivalent to one divided by those progressively increasing positive integers – thus a diminishing fractional number that never quite reaches 0.)

Practice Problem 2-3 (Answer: $\Sigma w_i x_i / \Sigma w_i$.)

Practice Problem 2-4 (Answer: $(x^3)(x^2+3)=x^5 + 3x^3$. If you said x^6 instead of x^5, remember that exponents are added when multiplied, and multiplied when raised to exponential powers. Thus, the product of $(x^3)(x^2)= x^5$ but the product of $(x^3)^2=x^6$. If you find this confusing, think of examples using numbers instead of letters: 5^1 times 5^1 indicates multiplying 5 times 5, which is 5^2 rather than 5^1; similarly, 5^1 times 5^2 indicates multiplying 5 times 5 times 5, which is 5^3 rather than 5^2.)

Practice Problem 2-5 (Answer: It is a number larger than 0.05 and smaller than 0.10.)

Practice Problem 2-6 (Answer: Since division by zero is not a legal operation, x in this function can be any positive or negative real number value but not the value zero.)

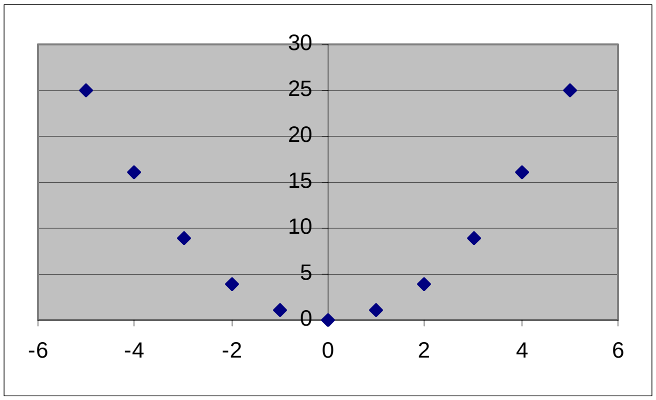
Practice Problem 2-7 (Answer: The index could be expressed as $Q = w/h^2$ for weight in kilograms and height in meters. Weight in kilograms equals weight in pounds divided by 2.2; height in meters equals height in inches divided by 39.4; therefore $Q = 39.4^2 w/2.2h^2 = 705.6w/h^2$ for weight in pounds and height in inches.)

Practice Problem 3-1 Answers:
a. $[(2+4)^2]^{1/2} = (6^2)^{1/2} = (36)^{1/2} = 6$
b. $(2)(4)/2 + (-2)(-4) = 4+8 = 12$
c. $2^{-3} = 1/(2^3) = 1/8$

Practice Problem 3-2 Answers:
a. $x(x+y)/y$
b. $y+(y-1) + \ldots +(-3)$ or could also be written as ${}_{i=0}{}^{y+3}\Sigma(y-i)$ or in other ways too
c. $2x = x^2$

Practice Problem 3-3 (Answer: This is just like the drug response time example above. We know two things about the age relationship: $P=2C$ and $P-C=30$. Solving both equations for P leads to the equivalency 2C=30+C, so the parent's age P must be 60 and the child's age C must be 30 years.)

Practice Problem 3-4 Answer: The graph would be a curve, something like this



Practice Problem 3-5 (Answer: There are 15! permutations. If each permutation took 5 minutes to draw on a chart, that means picturing 1,307,674,368,000 different arrangements at 5 minutes each! If you work nonstop, don't plan on finishing for a few centuries.)

B. Self-Assessment Questions

Higher-level courses in analytic techniques and statistical methods that you will take during a degree program probably are quite different from high school or undergraduate arithmetic or basic statistics courses you may have taken. They focus on higher-order learning objectives (ability to apply & explain, not just identify & define), building conceptual knowledge and comprehension rather than just memorization, and you will be expected to engage in discussion rather than learn passively through lectures. In order to succeed, you need to be able to speak the language that will be used. If you can answer questions like those that follow, then your numeracy skills are strong enough to take a higher-level course now. If you can't answer at least half the questions in each section correctly, then you should take advantage of numeracy self-study materials available before undertaking such analytic techniques courses.

Notation

1. Determine the value of x in each case.
   D. $x = (25)^{1/2}$
   E. $x = \sqrt{25}$
   F. $x^2 = 25$

(Answer (a) is 5, (b) is 5, (c) is ±5 not just +5)

2. Solve $-(x-2)(x-6) > 0$

(Answer $2<x<6$)

3. Is $(2x)^5$ equivalent to $2x^5$? Explain.

(Answer No. In the first expression, the base being raised is 2x but in the second it is just x being raised to the fifth power.)

4. Given the values x=2, 4, 6, 8, 10, for i=1 to n what is the value of Σx_i?

(Answer 30)

5. Given the values x=1, 2, 3, 4, 5, for i=1 to n sketch a graph of i vs. x_i

(Answer, a straight 45 degree line.)

6. Which of the following is a function and not an equation?
   - A. 32
   - B. $x + 32 = 0$
   - C. $32x + 16y = 7$
   - D. $x + 32 < 0$
   - E. $x + 32$

(Answer A is a constant; B & C are equations; D is an inequality statement, arguably a function & not an equation; E is a function but not an equation.)

Operations and Order of Operations

7. Solve $\sqrt{x+2} - 5 = 0$

(Answer 23)

8. Solve $\sqrt{x+2} + 5 = 0$

(Answer No real roots.)

9. Solve $[2^3][6 - 5(12 – 4 \times 5)]/3^3$

(Answer 13.63)

Working with rates, ratios, proportions, odds, percentiles

10. Rosa invested $15,000 in two funds. One was risky but paid 11% interest. The other was safer but paid only 6% interest. Rosa made $1250 interest in the first year. How much money did she invest in each fund?
   - A. $10,000 at 11% and $5,000 at 6%
   - B. $7,000 at 11% and $8,000 at 6%

C. \$12,000 at 6% and \$3,000 at 11%
D. \$6,000 at 6% and \$9,000 at 11%
E. None of A, B, C, and D.

(Answer B)

11. In 1991, one preventable cause of death accounted for approximately 20% of deaths in Canadian men and approximately 20% of deaths in Canadian women. Approximately what percentage of deaths among all Canadians could be attributed to that cause in 1991?

(Answer 20%)

12. If the probability of dying during a risky surgery is 80%, then the odds against survival are ??:??

(Answer 80:20 or 4:1)

13. A market survey determined that 40% of teenagers watch television news, 30% read the newspaper, and 20% watch television news and read the newspaper. What is the probability that a randomly chosen teenager does not read the newspaper or watch television news?

A. 90%
B. 80%
C. 50%
D. 20%
E. 10%

(Answer C, which is the complement of the proportion who watch television news OR read the paper. That is the proportion who watch television (40%) plus the proportion who read the paper (30%) minus the proportion who watch AND read (20%), since we shouldn't count them twice, all subtracted from the total 100%.)

14. The University of British Columbia is located in an earthquake region, so we often hear about numbers on the Richter Scale. It is described as a logarithmic scale because each number on that earthquake scale represents relative change in powers of 10. A magnitude 3 earthquake is how much more powerful than a magnitude 1 earthquake?

A. 3 times as powerful
B. 10 times as powerful
C. 30 times as powerful
D. 100 times as powerful.
E. 10^3 times as powerful

(Answer D)

Combinations, Permutations, and Factorials

15. There are five empty desks in a classroom. In how many ways can two students be assigned to these seats?

A. 5!
B. 2!
C. 52
D. ${}_5P_2$
E. ${}_5C_2$

(Answer D)

16. Simplify:

$$\frac{n(n-1)(n-2)(n-3)\ldots(2)(1)}{(n-2)(n-3)\ldots(2)(1)}$$

(Answer $n(n-1)$ *or* n^2-n*)*